Basic relations of a unified theory of electrodynamics, quantum mechanics, and gravitation*

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A first consistent variational principle of electrodynamics, quantum mechanics, and gravitation yields the basic equations of a unified scalar-vector-tensor theory which deals with extended particles of variable shape. It is compatible to solid principles of classical physics and, in particular, to special and general relativity. There is a direct way leading to a natural quantization of Maxwell's equations, to the Klein-Gordon equation, or to Einstein's equations including a quantum energy-momentum tensor of matter now. As an exemplification, Planck's fundamental energy-frequency relation is derived for transitions of charged particles in an external electric field and verified by calculating the energy integral of the H-atom.

A) Notation – Besides the well-known Landau-Lifshitz[1] notation there are used some self-explanatory abbreviations and the natural constants $e, c, \hbar, \kappa \equiv 8\pi G/c^4$. Capital italic letters like in A_i will denote macroscopically smoothed quantities in contrast to their microscopic counterparts like in a_i^{K}. K is a particle index, whereas $\bar{\mathrm{K}}$ implies an immediate summation over all particles *except for* K. A tilde like in $\tilde{s}_i^{\mathrm{K}} \equiv s_i^{\mathrm{K}} + e_{\mathrm{K}}/c \cdot a_i^{\bar{\mathrm{K}}}$ indicates a combination of components. $m_{\mathrm{K}}, e_{\mathrm{K}}, \mu_{\mathrm{K}}, \rho_{\mathrm{K}}$ are rest mass m_0, charge e_0, and their densities μ_0, ρ_0 with respect to particle K.

B) The key to a necessary extension of electrodynamics – Because of the local relativity of simultaneity some prototypes of quantum mechanics such as particles in a box, rotator, or oscillator turn out to be dynamic paradoxa[2] of special relativity theory (SRT). To avoid contradictions, it seems necessary to *presuppose* an energy-momentum postulate with respect to arbitrarily chosen inertial systems

$$\int T_i^{\,0} \mathrm{d}V = P_i \overset{!}{=} \mathit{constant}_i \,|_{\mathrm{SRT}}, \tag{1}$$

i.e. the conservation laws

$$\partial_k T_i^{\,k} = 0\,. \tag{2}$$

Therefore, neglecting internal gravitation, particles together with their carrier of interaction have to fulfill (2) within freely falling local frames. Since any objects are composed of charged particles, each physical process is subject to electrodynamics. At a closer look*, inserting the conventional energy-momentum tensor

$$T^{\,k}_{i\,(\mathrm{conventional})} \equiv \Big(- F_{il}F^{kl} + \frac{1}{4}\,\delta_i^k F_{rl}F^{rl} \Big) + \mu_0 c^2 U_i U^k, \tag{3}$$

phenomenologically combined of two independent parts so far, equation (2) implies a 'classical' relation between current density and four-potential

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$$J^{\,l} := \rho_0 U^{\,l} \stackrel{!}{=} -\frac{e_0}{m_0 c} Q^2 \left(\partial^{\,l} S + \frac{e_0}{c} A^l\right) \tag{4}$$

as the key to a necessary extension of electrodynamics. The derivation of (4) is based on the simplest ansatz

$$\rho_0 := e_0 \frac{\mu_0}{m_0} \equiv e_0 Q^2, \tag{5}$$

where a scalar Q is defined, which in its microscopic form q_{K} determines the shape of particle K. This new shape scalar immediately refers to quantum mechanics.

C) A first consistent variational principle of electrodynamics, quantum mechanics, and gravitation – The basic relations of the unified scalar-vector-tensor theory are derived from the variational principle

$$\delta \int \Big(\check{\varPhi} + \sum_{\mathrm{K}} \hat{\varPhi}_{\mathrm{K}} \Big) \sqrt{-g}\, \mathrm{d}\varOmega \;=\; 0 \tag{6}$$

which is composed of the action densities

$$\hat{\varPhi}_{\mathrm{K}} \equiv \frac{1}{4} f^{\,rl}_{\mathrm{K}} f^{\bar{\mathrm{K}}}_{rl} - \frac{1}{2m_{\mathrm{K}}} q^2_{\mathrm{K}} \left(s^{\,l}_{\mathrm{K}} + \frac{e_{\mathrm{K}}}{c} a^{\,l}_{\bar{\mathrm{K}}}\right) \left(s^{\mathrm{K}}_l + \frac{e_{\mathrm{K}}}{c} a^{\bar{\mathrm{K}}}_l\right) + \frac{1}{2} m_{\mathrm{K}} c^2 q^2_{\mathrm{K}} - \frac{\hbar^2}{2m_{\mathrm{K}}} q^{\,l}_{\mathrm{K}} q^{\mathrm{K}}_l \tag{7}$$

of each particle K, wherein q^{K}_l, s^{K}_l are the partial derivatives $\partial_l q^{\mathrm{K}}$, $\partial_l s^{\mathrm{K}}$. The index $\bar{\mathrm{K}}$ (non-K) is defined by

$$..^{\bar{\mathrm{K}}} \equiv \sum_{\nu \neq \mathrm{K}} ..^{(\nu)}\,, \tag{8}$$

and Einstein's overall gravitational action density is

$$2\kappa \check{\varPhi} := G \equiv g^{um} g^{sv} g^{rw} \left(\varGamma_{v,ur} \varGamma_{w,ms} - \varGamma_{v,um} \varGamma_{w,sr}\right). \tag{9}$$

The expression (7) assigned to a single particle K includes a familiar looking scalar of the electromagnetic field which, however, means field products of different particles only. This prevents from any self energy or renormalization problems, though the actual reason for this approach is the impossibility otherwise to get a consistent energy-momentum tensor satisfying the conservation laws (2). Modified correspondingly, a new model of electromagnetic waves seems compatible to photons now.

D) The quantized Maxwell equations – While the 1st pair is fulfilled as an identity by definition, $f^{\mathrm{K}}_{ik;l} + f^{\mathrm{K}}_{kl;i} + f^{\mathrm{K}}_{li;k} \equiv 0$, the variation of (6) with respect to *all* microscopic electromagnetic potentials a_l yields the 2nd pair in the form

$$f^{\,rl}_{\mathrm{K}\,;r} \equiv j^{\,l}_{\mathrm{K}} = -\frac{e_{\mathrm{K}}}{m_{\mathrm{K}} c} q^2_{\mathrm{K}} \left(s^{\,l}_{\mathrm{K}} + \frac{e_{\mathrm{K}}}{c} a^{\,l}_{\bar{\mathrm{K}}}\right), \tag{10}$$

where the electromagnetic potential of the respective particle K does not appear itself, but the potentials of all other particles $\bar{\mathrm{K}}$ instead.

E) The covariant real wave equation – Variation of (6) with respect to the shape scalar q_{K} yields

$$\hbar^2 q^{\,;l}_{\mathrm{K}\,;l} = q_{\mathrm{K}} \left[\left(s^{\,l}_{\mathrm{K}} + \frac{e_{\mathrm{K}}}{c} a^{\,l}_{\bar{\mathrm{K}}}\right) \left(s^{\mathrm{K}}_l + \frac{e_{\mathrm{K}}}{c} a^{\bar{\mathrm{K}}}_l\right) - m^2_{\mathrm{K}} c^2\right]. \tag{11}$$

It is this variable shape of *extended particles* (and *all* fields) that implies indeterminism. In the formal approximation $\hbar \to 0$, however, equation (11) reduces to that of Hamilton-Jacobi for charged *mass points* in a predefined electromagnetic field.

F) Continuity equation for rest mass and charge – Variation of (6) with respect to the microscopic action scalar $s_{\rm K}$ results in the covariant continuity equation

$$j^{\,l}_{{\rm K};l} = 0\,. \tag{12}$$

This continuity equation holds not only for the charge density but for the rest-mass density, too, since the latter is presupposed to be directly proportional to the charge density in (5). It is of decisive importance that (12) also results from (10).

G) The Einstein equations with a quantum energy-momentum tensor of matter – Variation of (6) with respect to the fundamental tensor g_{ik} yields

$$E_{ik} \equiv R_{ik} - \frac{1}{2} R g_{ik} = \kappa T_{ik}\,, \tag{13}$$

where the quantum tensor of matter and non-gravitational fields

$$T_{ik} \equiv \sum_{\rm K} T^{\rm K}_{ik} \tag{14}$$

is composed of

$$T^{\rm K}_{ik} = \tag{15}$$

$$-\frac{1}{2}\left(f^{\bar{\rm K}}_{il} f^{{\rm K}l}_{k} + f^{\bar{\rm K}}_{kl} f^{{\rm K}l}_{i} \right) + \frac{1}{m_{\rm K}} q^2_{\rm K} \tilde{s}^{\,\rm K}_i \tilde{s}^{\,\rm K}_k + \frac{\hbar^2}{m_{\rm K}} q^{\rm K}_i q^{\rm K}_k + \frac{1}{4} g_{ik} \left[f^{\,rl}_{\rm K} f^{\bar{\rm K}}_{rl} - \frac{\hbar^2}{m_{\rm K}} \left(q^2_{\rm K} \right)^{;l}_{\;;l} \right].$$

This derivation of Einstein's equations includes a consistent completion by a quantum energy-momentum tensor now. Its classical approximation – implying the 'geodesic' law of motion – has no longer to be added phenomenologically only.

H) Derivation of the complex Klein-Gordon equation – A mere substitution into (11), (12) of

$$q_{\rm K} \equiv \sqrt{\psi_{\rm K} \psi^*_{\rm K}}\,, \qquad s_{\rm K} \equiv -\frac{i\hbar}{2} \ln \left(\psi_{\rm K} / \psi^*_{\rm K} \right) \tag{16}$$

yields the Klein-Gordon equation

$$\frac{1}{\sqrt{-g}} \left(i\hbar \partial_l - \frac{e_{\rm K}}{c} a^{\bar{\rm K}}_l \right) \left[\sqrt{-g} \left(i\hbar \partial^l - \frac{e_{\rm K}}{c} a^{\,l}_{\rm K} \right) \right] \psi_{\rm K} = m^2_{\rm K} c^2 \psi_{\rm K} \tag{17}$$

in a *covariant* form. The complex quantum scalar $\psi_{\rm K}$ is normalized by the natural requirement that the integral of $j^0_{\rm K}$ equals $e_{\rm K}$ now. Only this unconventional normaliziation proves consistent with the energy-frequency relation for photons, too.

I) The energy-frequency relation – Thus, for example, using (14) and the basic relations above, a straightforward calculation of the energy of the H-atom results in

$$E^{(\rm stationary)}_{({\rm H})\infty} = \varepsilon_{{\rm e}\nu} + m_{\rm p} c^2\,, \tag{18}$$

where ∞ may indicate a fixed proton here. Therefore, what has been a mysterious quantum mechanical parameter $\varepsilon_{{\rm e}\nu}$ so far, actually turns out to be the true energy integral of the bound electron in stationary states ($\dot{q}_{\rm e} = 0$). With $\omega_{\mu\nu}$ the well-known oscillation frequency of the charge distribution during the transition $\mu \to \nu$, now $|\varepsilon_{{\rm e}\mu} - \varepsilon_{{\rm e}\nu}| = \hbar\omega_{\mu\nu}$ is *derived* as an energy quantum of radiation, i.e. of a photon.

References

1. Landau L. D., Lifschitz E. M.: Lehrbuch der theoretischen Physik; Berlin 1992
2. Ostermann, P.: Phys.u.D. 1 (1985) 23-37 (available from author's website, too)